ECL 4340

POWER SYSTEMS

Lecture 20
Economic Dispatch, Optimal Power Flow

Professor Kwang Y. Lee
Department of Electrical and
Computer Engineering

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Announcements

- Be reading Chapter 6, sections 6.12 & 6.13.
- Final is on Monday, December 12, 9:00 - 11:00 a.m. in Room 204

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Generator MW Limits

- Generators have limits on the minimum and maximum amount of power they can produce
- Often times the minimum limit is not zero. This represents a limit on the generator's operation with the desired fuel type
- Because of varying system economics usually many generators in a system are operated at their maximum MW limits.

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Lambda-Iteration with Gen Limits

In the Lambda-iteration method the limits are taken into account when calculating $\mathrm{P_{Gi}}(\lambda)$:

$$\text{if } \mathrm{P_{Gi}}(\lambda) > P_{Gi,\max} \text{ then } \mathrm{P_{Gi}}(\lambda) = P_{Gi,\max}$$

$$\text{if } \mathrm{P_{Gi}}(\lambda) < P_{Gi,\min} \text{ then } \mathrm{P_{Gi}}(\lambda) = P_{Gi,\min}$$

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Lambda-Iteration Example

Consider a three generator system with

$$IC_1(P_{G1}) = 15 + 0.02P_{G1} = \lambda \quad \$/\mathrm{MWh}$$

$$IC_2(P_{G2}) = 20 + 0.01P_{G2} = \lambda \quad \$/\mathrm{MWh}$$

$$IC_3(P_{G3}) = 18 + 0.025P_{G3} = \lambda \quad \$/\mathrm{MWh}$$

and with constraint $P_{G1} + P_{G2} + P_{G3} = 1000\,\mathrm{MW}$

Rewriting as a function of λ, $\mathrm{P_{Gi}}(\lambda)$, we have

$$\mathrm{P_{G1}}(\lambda) = \frac{\lambda - 15}{0.02} \qquad \mathrm{P_{G2}}(\lambda) = \frac{\lambda - 20}{0.01}$$

$$\mathrm{P_{G3}}(\lambda) = \frac{\lambda - 18}{0.025}$$

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Lambda-Iteration Example, cont'd

Pick λ^L & λ^H so that $\sum_{i=1}^{m} \mathrm{P_{Gi}}(\lambda^L) - 1000 < 0$ and

$$\sum_{i=1}^{m} \mathrm{P_{Gi}}(\lambda^H) - 1000 > 0$$

Try $\lambda^L = 20$, then $\sum_{i=1}^{m} P_{Gi}(20) - 1000 =$

$$\boxed{\frac{\lambda - 15}{0.02}} + \boxed{\frac{\lambda - 20}{0.01}} + \boxed{\frac{\lambda - 18}{0.025}} - 1000 = \underline{250 + 0 + 80} - 1000 = -670 \text{ MW}$$

Try $\lambda^H = 30$, then $\sum_{i=1}^{m} P_{Gi}(30) - 1000 = \underline{750 + 1000 + 480} - 1000 = 1230$

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Lambda-Iteration Gen Limit Example

In the previous three generator example assume the same cost characteristics but also with limits

$0 \le P_{G1} \le 300$ MW $\quad 100 \le P_{G2} \le 500$ MW

$200 \le P_{G3} \le 600$ MW

With limits we get

$$\sum_{i=1}^{m} P_{Gi}(20) - 1000 = P_{G1}(20) + P_{G2}(20) + P_{G3}(20) - 1000$$

$$= \underline{250 + 100 + 200} - 1000 = -450 \text{ MW (compared to } -670\text{MW)}$$

$$\sum_{i=1}^{m} P_{Gi}(30) - 1000 = \underline{300 + 500 + 480} - 1000 = 280 \text{ (vs 1230)}$$

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Lambda-Iteration Example, cont'd

Without the limits, continue iterating until $\left|\lambda^{H} - \lambda^{L}\right| < 0.05$

The solution value of λ, λ^*, is 23.53 \$/MWh

Once λ^* is known we can calculate the P_{Gi}

$$P_{G1}(23.5) = \frac{23.53 - 15}{0.02} = 426 \text{ MW}$$

$$P_{G2}(23.5) = \frac{23.53 - 20}{0.01} = 353 \text{ MW}$$

$$P_{G3}(23.5) = \frac{23.53 - 18}{0.025} = 221 \text{ MW}$$

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Lambda-Iteration Limit Example, cont'd

With the limits, we continue iterating until the convergence condition is satisfied. With limits the final solution of λ, is 24.43 \$/MWh (compared to 23.53 \$/MWh without limits). The presence of limits will always cause λ to either increase or remain the same.

Final solution is

$P_{G1}(24.43) = 300$ MW, compared to 426 MW

$P_{G2}(24.43) = 443$ MW, compared to 353 MW

$P_{G3}(24.43) = 257$ MW, compared to 221 MW

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Back of Envelope Values

- Often times, incremental costs can be approximated by a constant value:
  - \$/MWhr = fuelcost * heatrate + variable O&M
  - Typical heatrate for a coal plant is 10, modern combustion turbine is 10, combined cycle plant is 7 to 8, older combustion turbine 15.
  - Fuel costs (\$/MBtu) are quite variable, with current values around 1.5 for coal, 4 for natural gas, 0.5 for nuclear, probably 10 for fuel oil.
  - Hydro, solar and wind costs tend to be quite low, but for these sources the fuel is free but limited.

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Inclusion of Transmission Losses

- The losses on the transmission system are a function of the generation dispatch. In general, using generators closer to the load results in lower losses
- This impact on losses should be included when doing the economic dispatch
- Losses can be included by slightly rewriting the Lagrangian:

$$\mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda) = \sum_{i=1}^{m} C_i(P_{Gi}) + \lambda(P_D + \underline{P_L(P_G)} - \sum_{i=1}^{m} P_{Gi})$$

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Impact of Transmission Losses

This small change then impacts the necessary conditions for an optimal economic dispatch

$$\mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda) = \sum_{i=1}^{m} C_i(P_{Gi}) + \lambda(P_D + \underline{P_L(P_G)} - \sum_{i=1}^{m} P_{Gi})$$

The necessary conditions for a minimum are now

$$\frac{\partial \mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda)}{\partial P_{Gi}} = \frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right) = 0$$

$$P_D + \underline{P_L(P_G)} - \sum_{i=1}^{m} P_{Gi} = 0$$

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Impact of Transmission Losses

Solving each equation for λ we get

$$\frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}) = 0$$

$$\lambda = \frac{1}{\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right)} \frac{dC_i(P_{Gi})}{dP_{Gi}}$$

Define the penalty factor L_i for the i$^{\text{th}}$ generator

$$L_i = \frac{1}{\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right)}$$

The penalty factor at the slack bus is always unity!

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Impact of Transmission Losses

The condition for optimal dispatch with losses is then

$$L_1 \cdot IC_1(P_{G1}) = L_2 \cdot IC_2(P_{G2}) = L_m \cdot IC_m(P_{Gm}) = \lambda$$

Since $L_i = \frac{1}{\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right)}$ if increasing P_{Gi} increases

the losses then $\frac{\partial P_L(P_G)}{\partial P_{Gi}} > 0 \rightarrow L_i > 1.0$

This makes generator i appear to be more expensive (i.e., it is penalized). Likewise $L_i < 1.0$ makes a generator appear less expensive.

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Calculation of Penalty Factors

Unfortunately, the analytic calculation of L_i is somewhat involved. The problem is a small change in the generation at P_{Gi} impacts the flows and hence the losses throughout the entire system.

However, using a power flow you can approximate this function by making a small change to P_{Gi} and then seeing how the losses change:

$$\frac{\partial P_L(P_G)}{\partial P_{Gi}} \approx \frac{\Delta P_L(P_G)}{\Delta P_{Gi}} \qquad L_i \approx \frac{1}{1 - \frac{\Delta P_L(P_G)}{\Delta P_{Gi}}}$$

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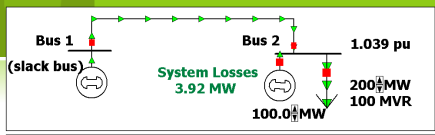
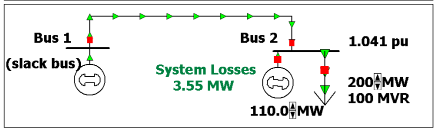
Two Bus Penalty Factor Example





$$\frac{\partial P_L(P_G)}{\partial P_{G2}} = -0.0387 \qquad \frac{\Delta P_L(P_G)}{\Delta P_{Gi}} = \frac{-0.37MW}{10MW} = -0.037$$

$$L_2 = 0.9627 \qquad L_i \approx \frac{1}{1 - \dfrac{\Delta P_L(P_G)}{\Delta P_{Gi}}} \qquad L_2 \approx 0.9643$$

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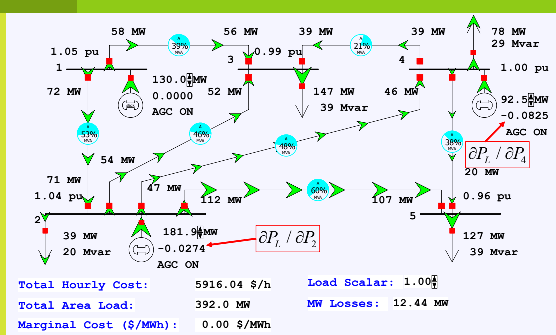
Example 6.22



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Optimal Power Flow (OPF)

- OPF functionally combines the power flow with economic dispatch
- Minimize cost function, such as operating cost, considering realistic equality and inequality constraints
- Equality constraints
  - bus real and reactive power balance
  - generator voltage setpoints
  - area MW interchange

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OPF, cont'd

- Inequality constraints
  - transmission line/transformer/interface flow limits
  - generator MW limits
  - generator reactive power capability curves
  - bus voltage magnitudes (not yet implemented in Simulator OPF)
- Available Controls
  - generator MW outputs
  - transformer taps and phase angles

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Two Example OPF Solution Methods

- Non-linear approach using Newton's method
  - handles marginal losses well, but is relatively slow and has problems determining binding constraints
- Linear Programming
  - fast and efficient in determining binding constraints but can have difficulty with marginal losses.
  - used in PowerWorld Simulator

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LP OPF Solution Method

- Solution iterates between
  - solving a full ac power flow solution
    - enforces real/reactive power balance at each bus
    - enforces generator reactive limits
    - system controls are assumed fixed
    - takes into account nonlinearities
  - solving a primal LP
    - changes system controls to enforce linearized constraints while minimizing cost

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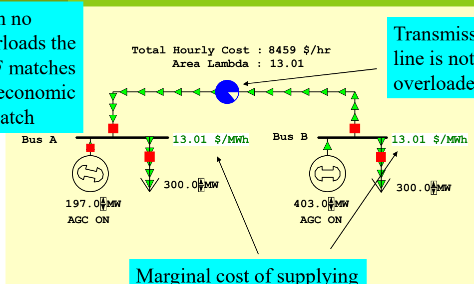
Two Bus with Unconstrained Line

With no overloads the OPF matches the economic dispatch

Transmission line is not overloaded



Marginal cost of supplying power to each bus (locational marginal costs)

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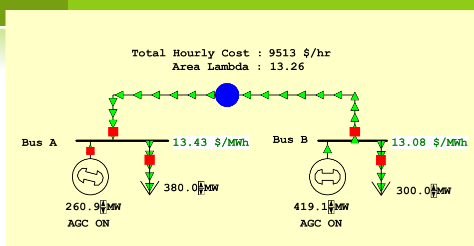
Two Bus with Constrained Line



With the line loaded to its limit, additional load at Bus A must be supplied locally, causing the marginal costs to diverge.

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Three Bus (B3) Example

- Consider a three-bus case (bus 1 is system slack), with all buses connected through 0.1 pu reactance lines, each with a 100 MVA limit
- Let the generator marginal costs be
  - Bus 1: 10 $ / MWhr; Range = 0 to 400 MW
  - Bus 2: 12 $ / MWhr; Range = 0 to 400 MW
  - Bus 3: 20 $ / MWhr; Range = 0 to 400 MW
- Assume a single 180 MW load at bus 2

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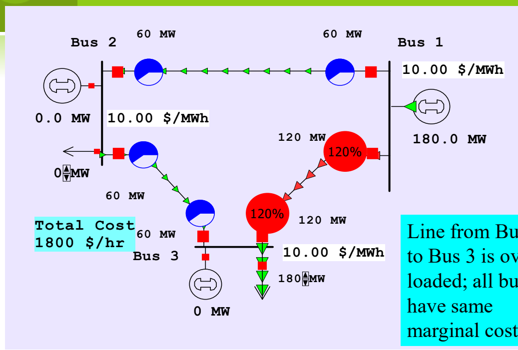
B3 with Line Limits NOT Enforced



Line from Bus 1 to Bus 3 is over-loaded; all buses have same marginal cost

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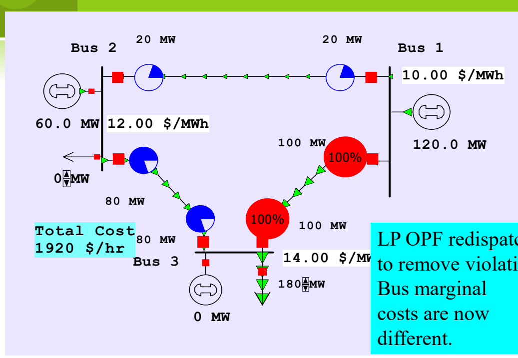
B3 with Line Limits Enforced



LP OPF redispatches to remove violation. Bus marginal costs are now different.

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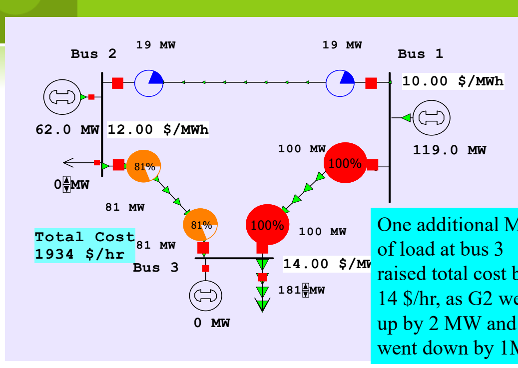
Verify Bus 3 Marginal Cost



One additional MW of load at bus 3 raised total cost by 14 $/hr, as G2 went up by 2 MW and G1 went down by 1MW

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Why is bus 3 LMP = $14 /MWh

- All lines have equal impedance. Power flow in a simple network distributes inversely to impedance of path.
  - For bus 1 to supply 1 MW to bus 3, 2/3 MW would take direct path from 1 to 3, while 1/3 MW would "loop around" from 1 to 2 to 3.
  - Likewise, for bus 2 to supply 1 MW to bus 3, 2/3MW would go from 2 to 3, while 1/3 MW would go from 2 to 1 to 3.

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Why is bus 3 LMP $ 14 / MWh, cont'd

- With the line from 1 to 3 limited, no additional power flows are allowed on it.
- To supply 1 more MW to bus 3 we need
  - PG1 + PG2 = 1 MW
  - 2/3 PG1 + 1/3 PG2 = 0; (no more flow on 1-3)
- Solving requires we up PG2 by 2 MW and drop PG1 by 1 MW -- a net increase of $14.

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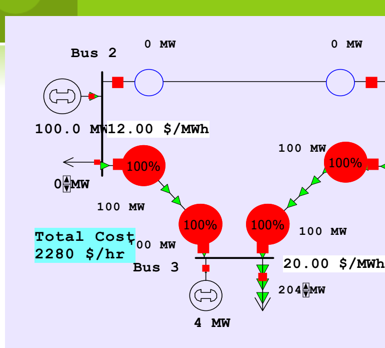
Both lines into Bus 3 Congested



For bus 3 loads above 200 MW, the load must be supplied locally. Then what if the bus 3 generator opens?

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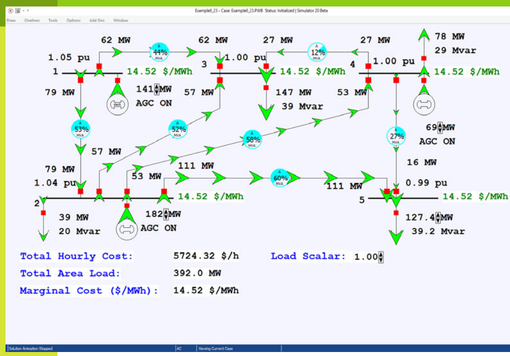
Example 6_23 Optimal Power Flow



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